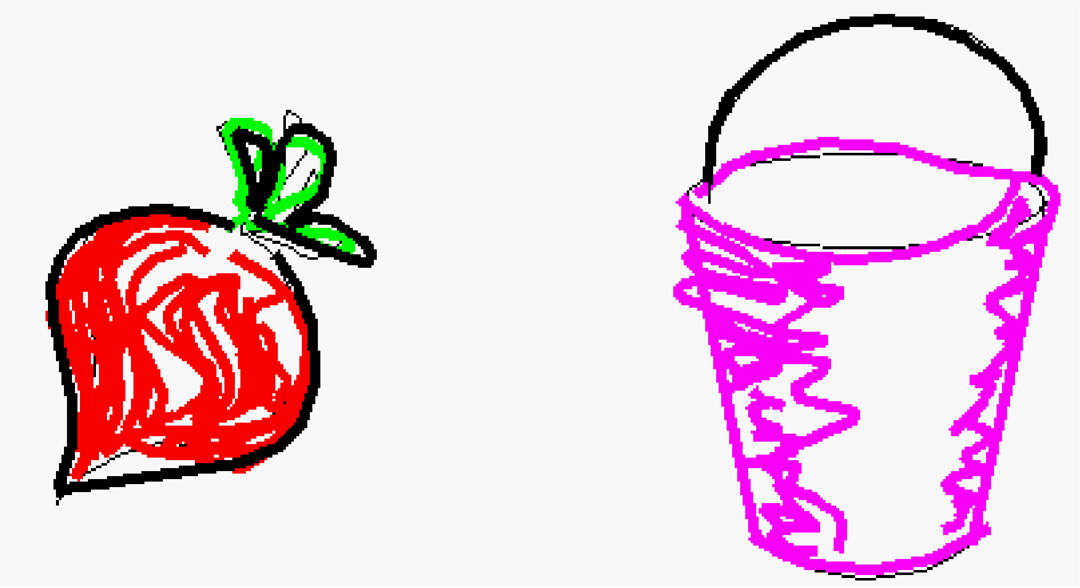
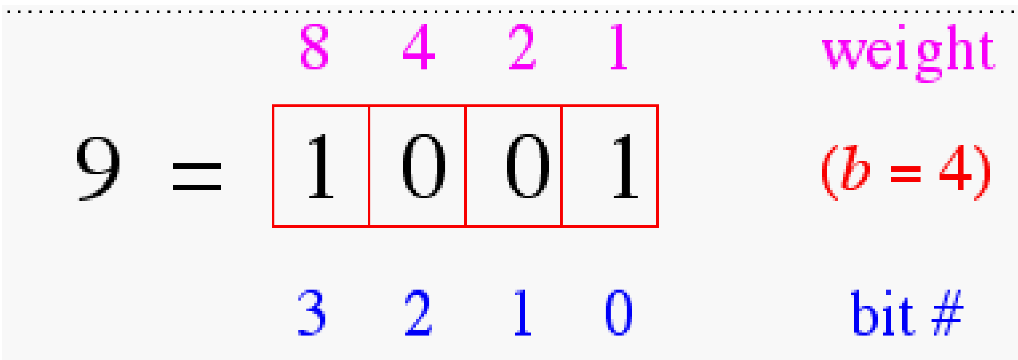
More Sorting

- radix sort
- bucket sort
- in-place sorting
- how fast can we sort?



Radix Sort

- Unlike other sorting methods, radix sort considers the structure of the keys
- Assume keys are represented in a base M number system (M = radix), i.e., if M = 2, the keys are represented in binary



Sorting is done by comparing bits in the same position

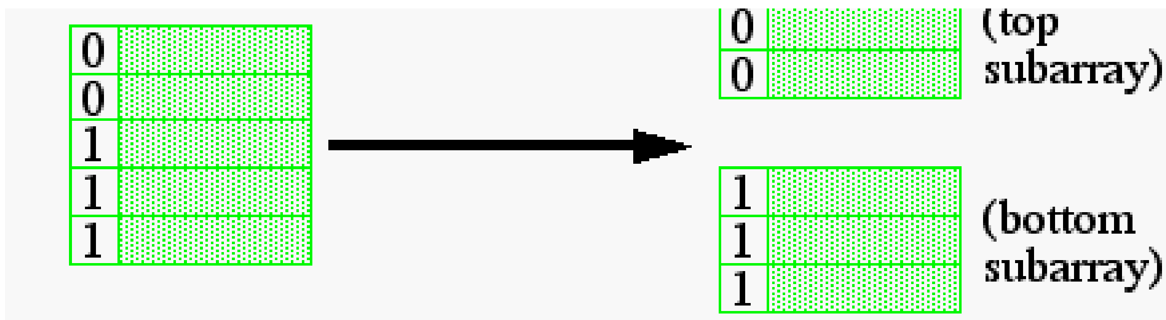
Extension to keys that are alphanumeric strings

Radix Exchange Sort

Examine bits from left to right:

1. Sort array with respect to leftmost bit

2. Partition array



3. Recursion

recursively sort top subarray, ignoring leftmost bit

recursively sort bottom subarray, ignoring leftmost bit

Time to sort n b-bit numbers: ***O(b n)***

Radix Exchange Sort

How do we do the sort from the previous page? Same idea as partition in Quicksort.

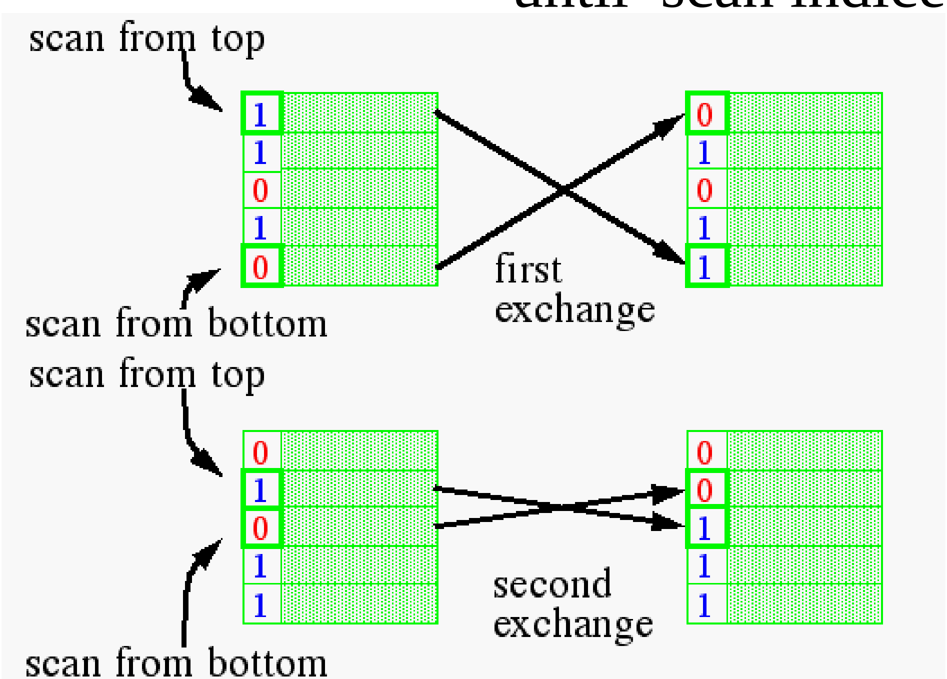
repeat

scan top-down to find key starting with 1;

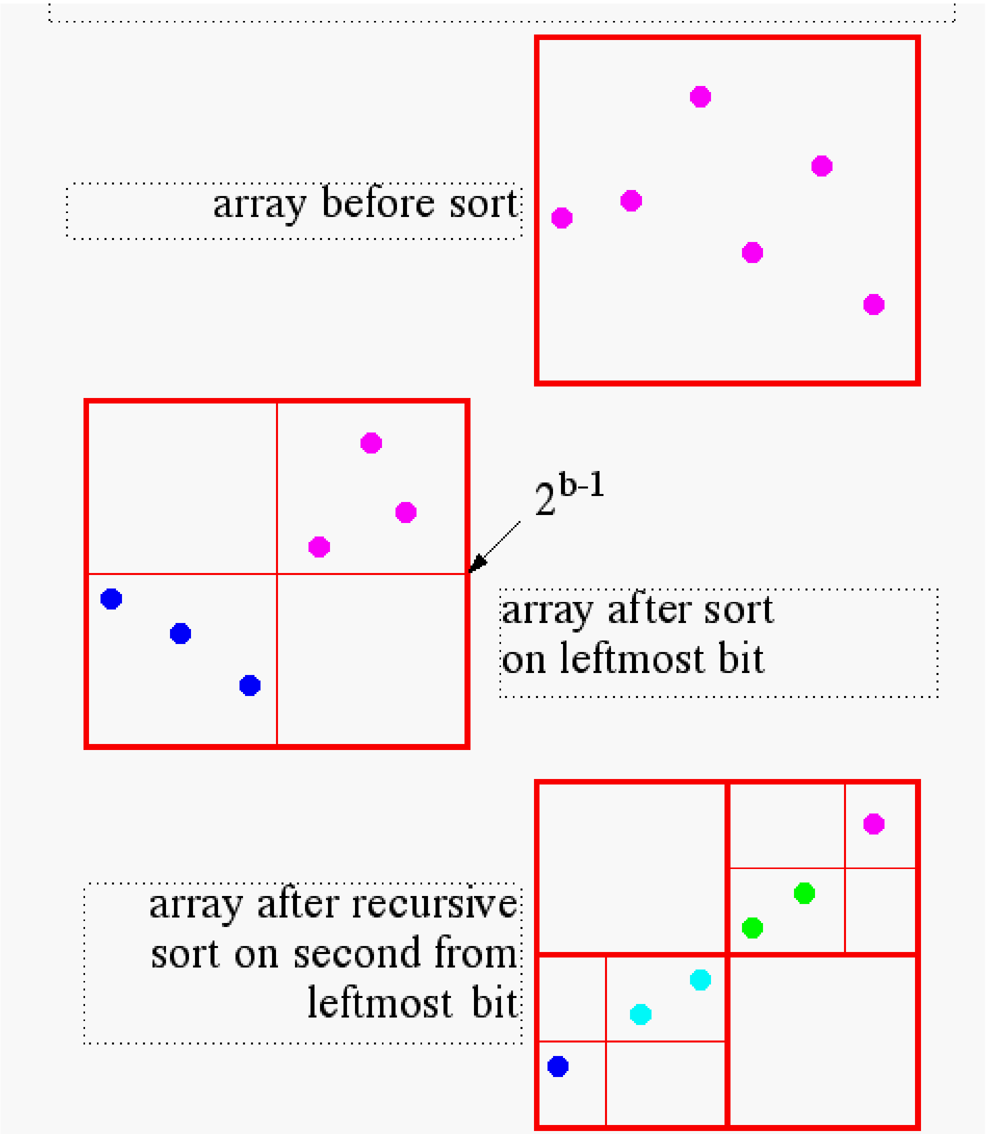
scan bottom-up to find key starting with 0;

exchange keys;

until scan indices cross;



Radix Exchange Sort



Radix Exchange Sort vs. Quicksort

Similarities

both partition array

both recursively sort sub-arrays

Differences

Method of partitioning

radix exchange divides array based on greater than or less than 2^{b-1}

quicksort partitions based on greater than or less than some element of the array

Time complexity

Radix exchange O (bn)

Quicksort average case O (n log n)

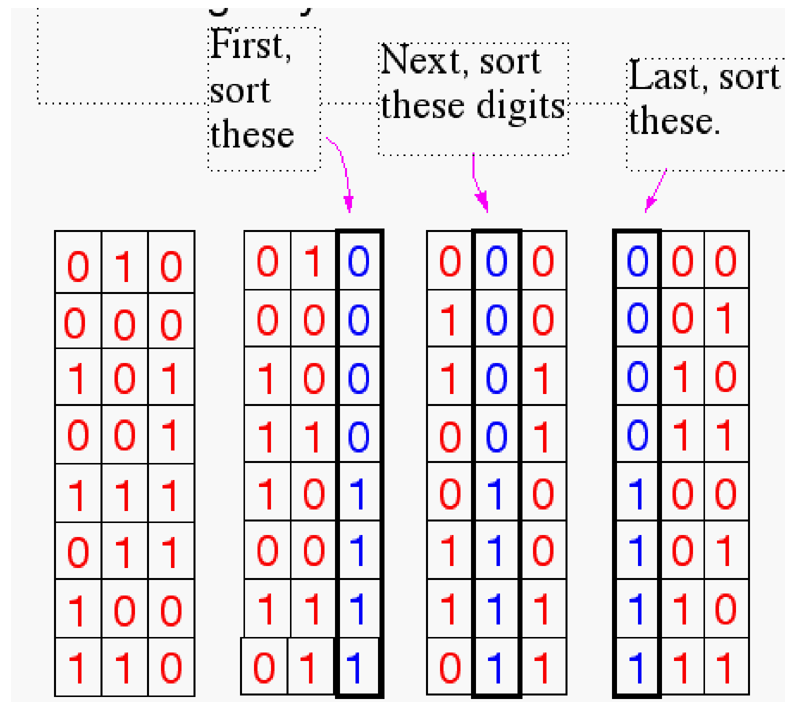
Straight Radix Sort

Examines bits from right to left

for k := 0 to b-1

sort the array in a stable way,

looking only at bit k



Note order of these bits after sort.

What is "sort in a stable way"!!!

In a stable sort, the initial relative order of equal keys is unchanged. For example, observe the first step of the sort from the previous page:

0	1	0
0	0	0
1	0	1
0	0	1
1	1	1
0	1	1
1	0	0
1	1	0

→

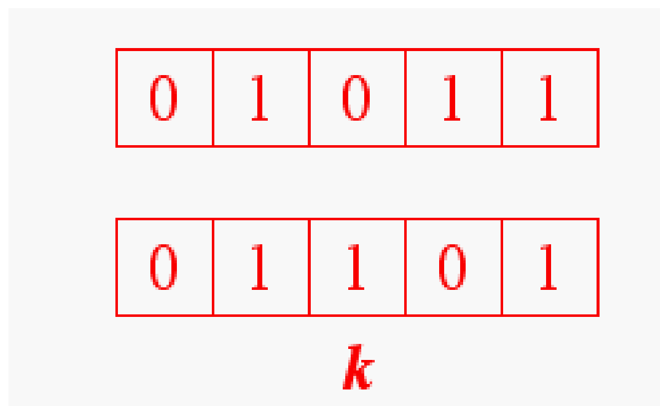
0	1	0
0	0	0
1	0	0
1	1	0
1	0	1
0	0	1
1	1	1
0	1	1

Note that the relative order of those keys ending with 0 is unchanged, and the same is true for elements ending in 1

The Algorithm is Correct (right?)

We show that any two keys are in the correct relative order at the end of the algorithm

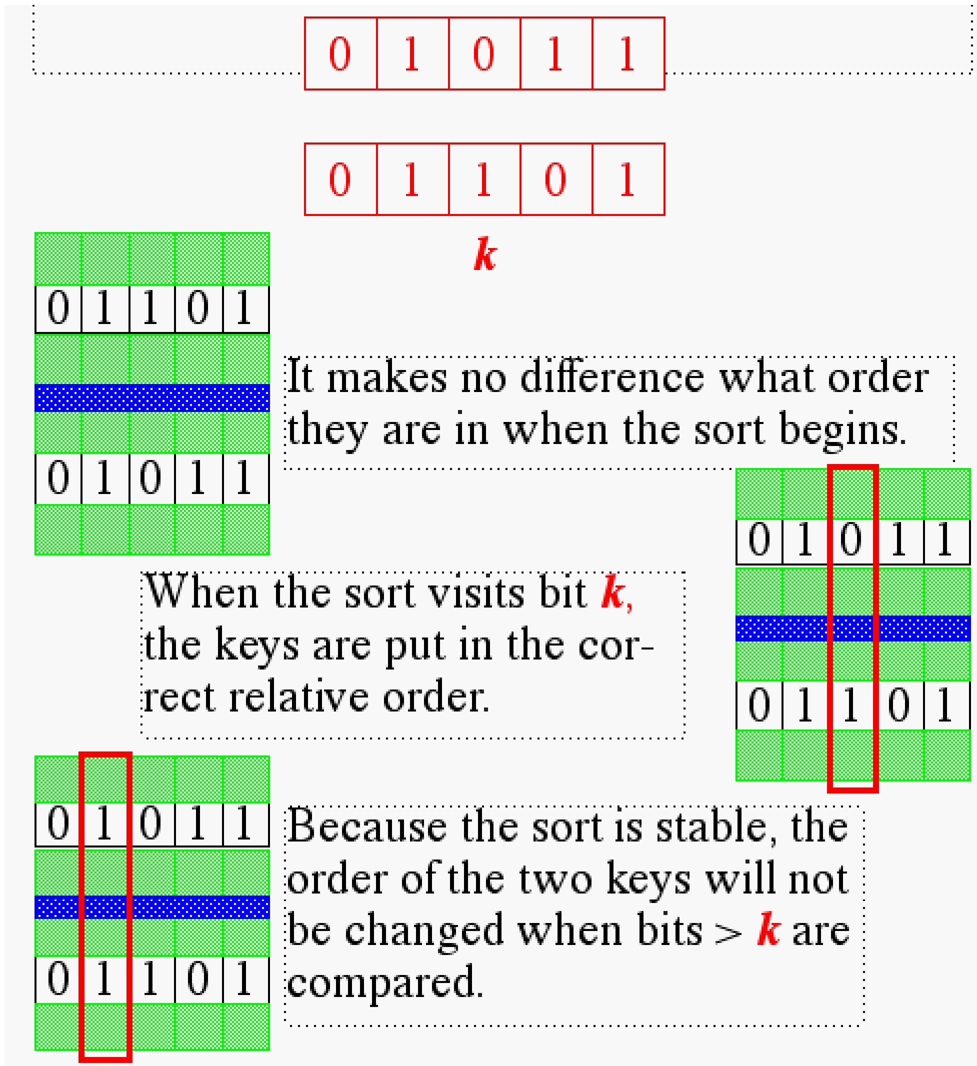
Given two keys, let ***k*** be the leftmost bit-position where they differ



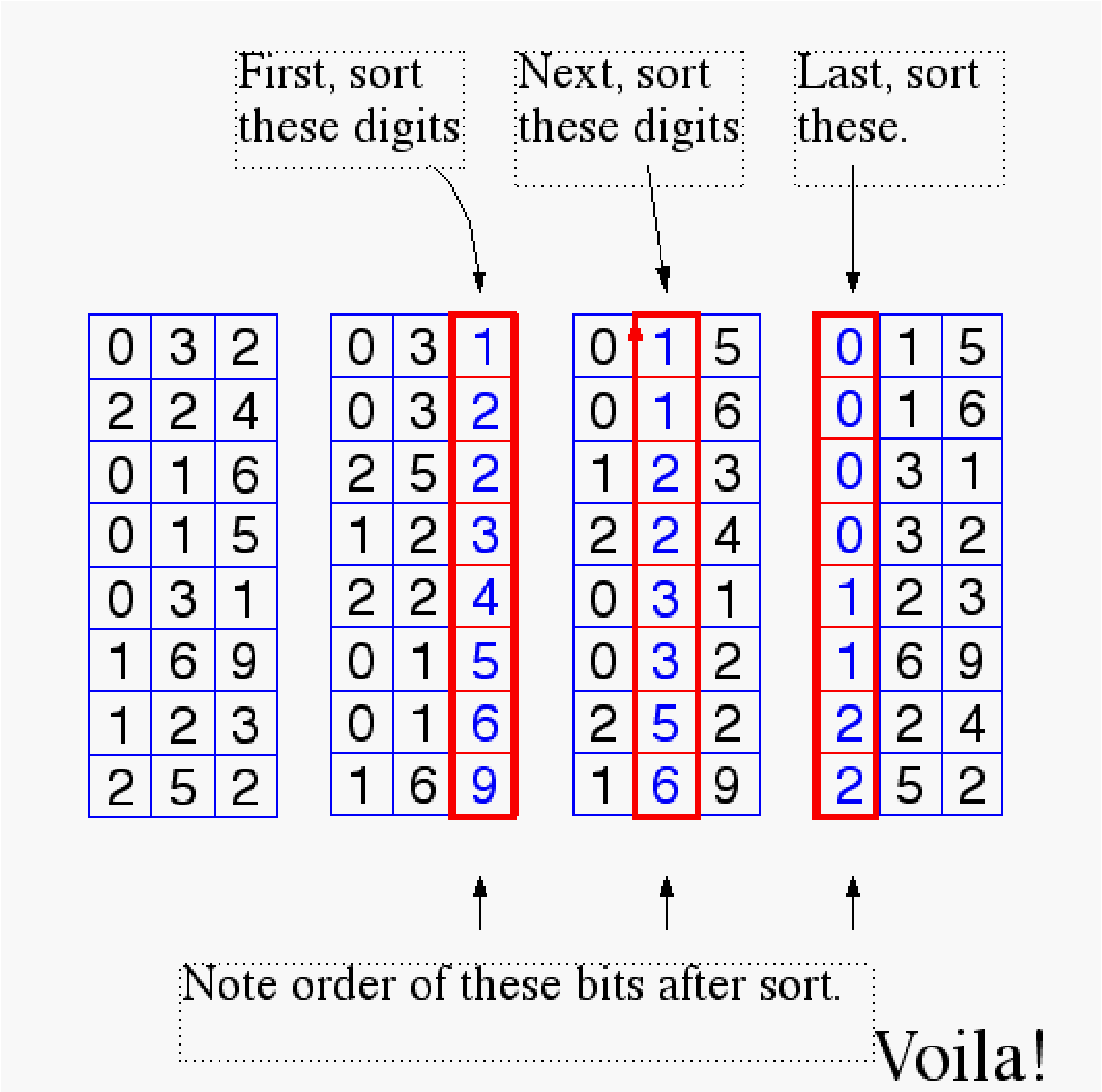
At step ***k*** the two keys are put in the correct relative order
Because of ***stability***, the successive steps do not change the relative order of the two keys

For Instance,

Consider a sort on an array with these two keys:



Radix sorting applied to decimal numbers



Straight Radix Sort Time Complexity

for $k = 0$ to $b - 1$

sort the array in a stable way, looking only at bit k

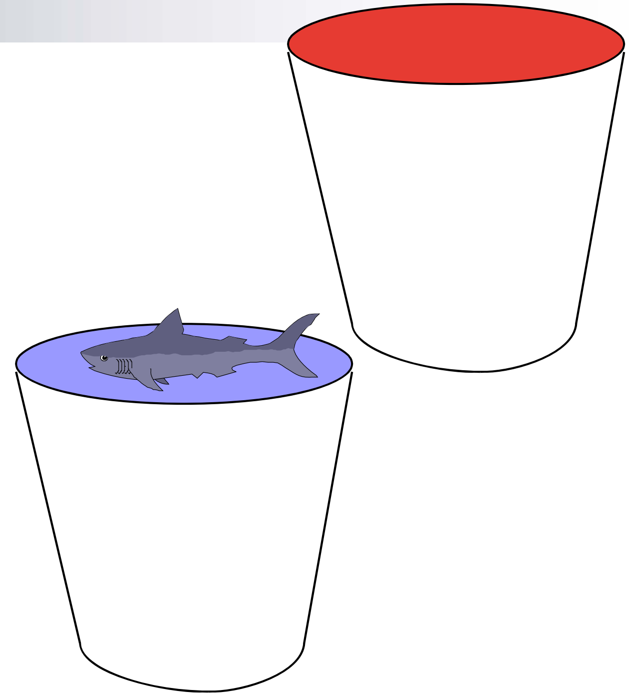
Suppose we can perform the stable sort above in $O(n)$ time. The total time complexity would be

O(bn)

As you might have guessed, we can perform a stable sort based on the keys' k^{th} digit in $O(n)$ time.

The method, you ask? Why it's Bucket Sort, of course.

Bucket Sort



BASICS:

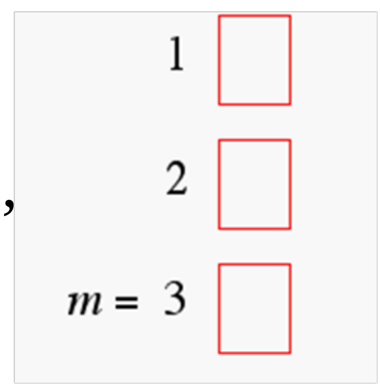
- **n numbers**
- **Each number ∈ {1, 2, 3, ... m}**
- **Stable**
- **Time: O (n + m)**

For example, m = 3 and our array is:

2	1	3	1	2

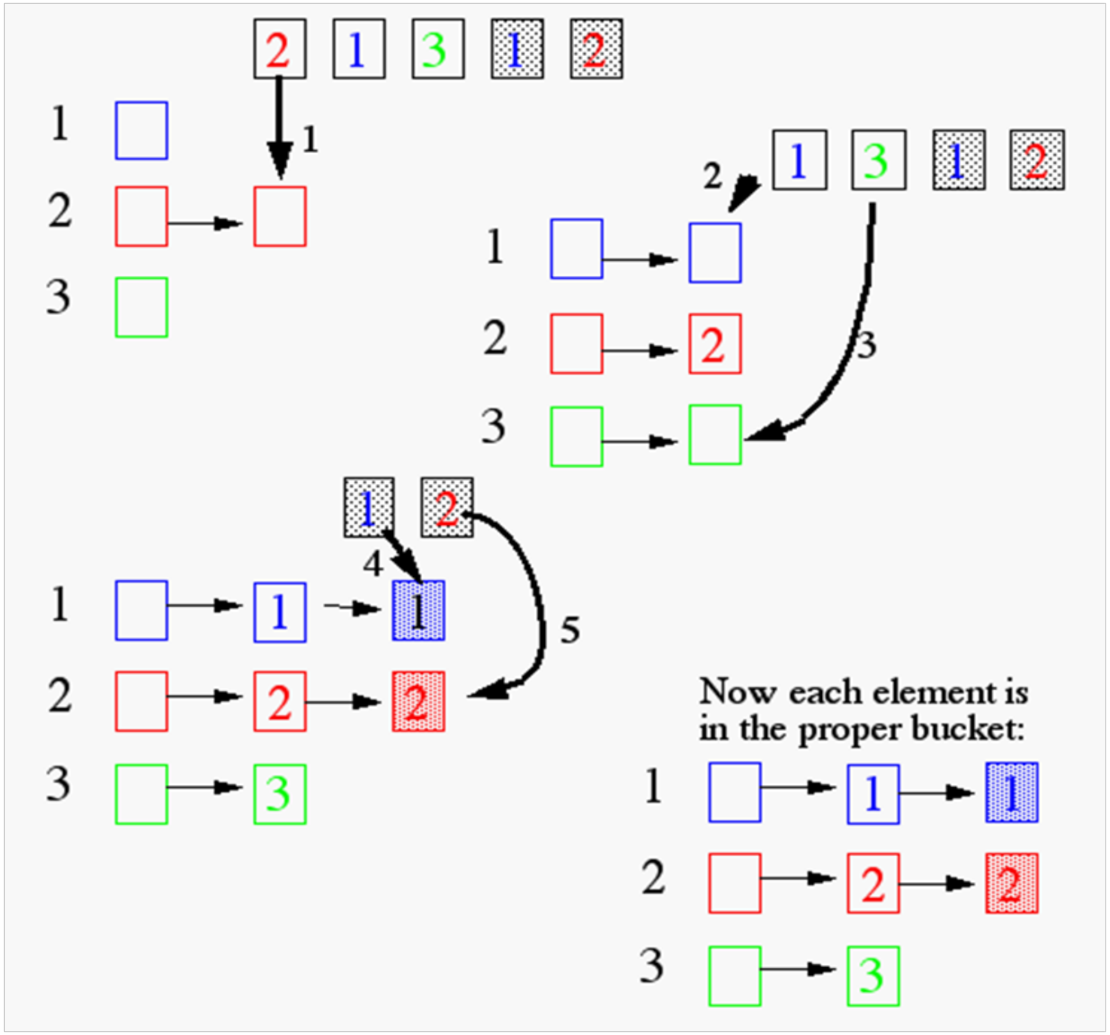
(note that there are two “2”s and two “1”s)

First, we create M “buckets”



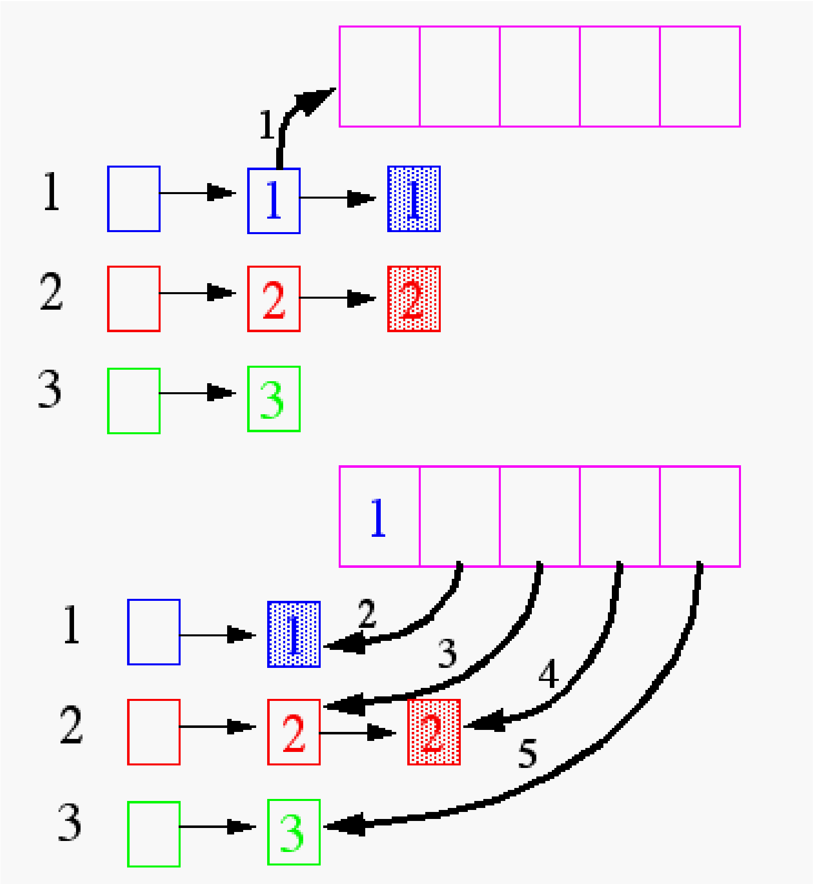
Bucket Sort

Each element of the array is put in one of the m “buckets”

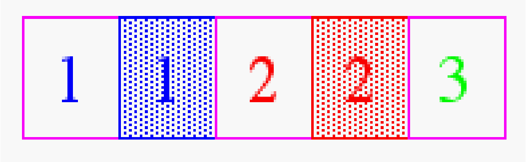


Bucket Sort

Now, pull the elements from the buckets into the array



At last, the sorted array (sorted in a stable way):



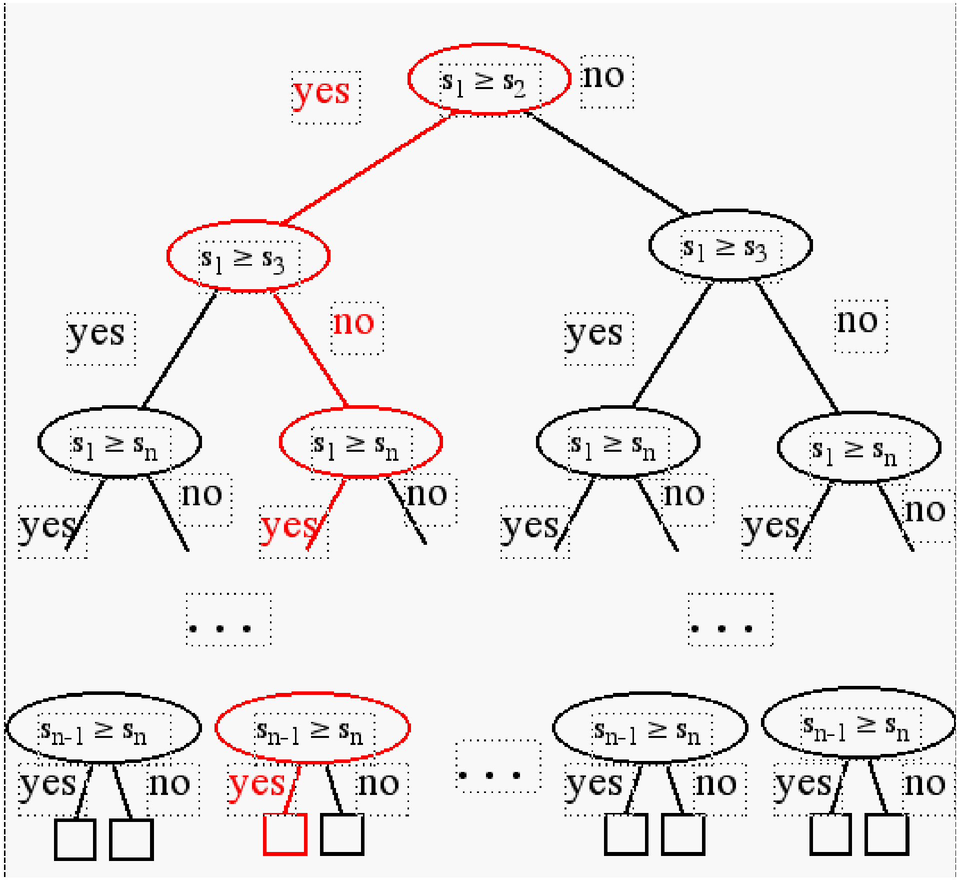
In-Place Sorting

- A sorting algorithm is said to be in-place if
  - it uses no auxiliary data structures (however, O(1) auxiliary variables are allowed)
  - it updates the input sequence only by means of operations replaceElement and swapElements
- Which sorting algorithms seen can be made to work in place?

bubble-sort	Y
selection-sort	
insertion-sort	
heap-sort	
merge-sort	
quick-sort	
radix-sort	
bucket-sort	

Lower Bd. for Comparison Based Sorting

- internal node: comparison
- external node: permutation
- algorithm execution: root-to-leaf path



How Fast Can We Sort?

- How Fast Can We Sort?
- Proposition: The running time of any comparison-based algorithm for sorting an n-element sequence S is $\Omega(n \log n)$.
- **Justification:** The running time of a comparison-based sorting algorithm must be equal to or greater than the depth of the decision tree T associated with this algorithm.

How Fast Can We Sort? (2)

- Each internal node of T is associated with a comparison that establishes the ordering of two elements of S.
- Each external node of T represents a distinct permutation of the elements of S.
- Hence T must have at least n! external nodes which again implies T has a height of at least log(n!)
- Since n! has at least n/2 terms that are greater than or equal to n/2, we have: $\log(n!) \geq (n/2) \log(n/2)$
- Total Time Complexity: $\Omega(n \log n)$.