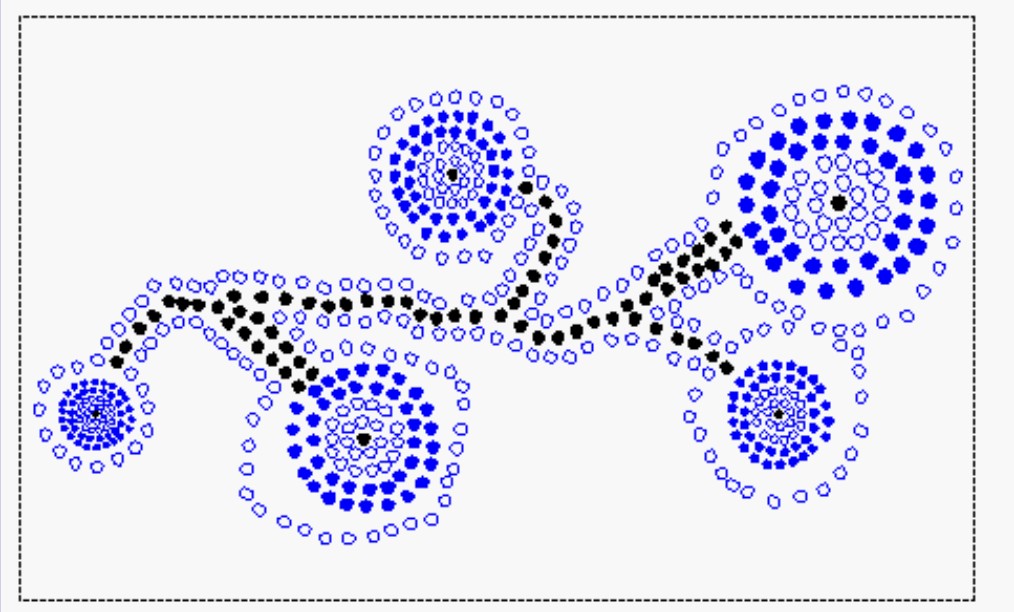
Priority Queues

- Scheduling example
- The priority queue ADT
- Implementing a priority queue with a sequence
- Binary Heaps
- Insertion in a Heaps and Heapify



Scheduling

- In a multi-user computer system, multiple users submit jobs to run on a single processor.
- We assume that the time required by each job is known in advance. Further, jobs can be preempted (stopped and resumed later)
- One policy which minimizes the average waiting time is SRPT (shortest remaining processing time).
- The processor schedules the job with the smallest remaining processing time.
- If while a job is running a new job arrives with processing time less than the remaining time of current job, the current job is preempted.

Data Structure for SRPT

- We need to maintain the remaining processing time of the unfinished jobs at any point in time.
- We need to find the job with the shortest remaining processing time.
- When a job finishes we should remove it from our collection.
- When a new job arrives we need to add it to the collection.

Priority Queues

- A priority queue is an *ADT(abstract data type)* for maintaining a set S of elements, each with an associated value called priority
- A PQ supports the following operations
  - **Insert**(x) insert element x in set S ($S \leftarrow S \cup \{x\}$)
  - **Minimum**() returns the element of S with smallest priority.
  - **Delete-min**() returns and removes the element of S with smallest priority.

Priorities and Total Order Relations

- A Priority Queue ranks its elements by **priority.**
- Every element has a priority. Priorities are not necessarily unique and are totally ordered.
- Total Order Relation, denoted by ≤

  Reflexive: k ≤ k

  Antisymetric: if k1 ≤ k2 and k2 ≤ k1, then k1 ≤ k2

  Transitive: if k1 ≤ k2 and k2 ≤ k3, then k1 ≤ k3

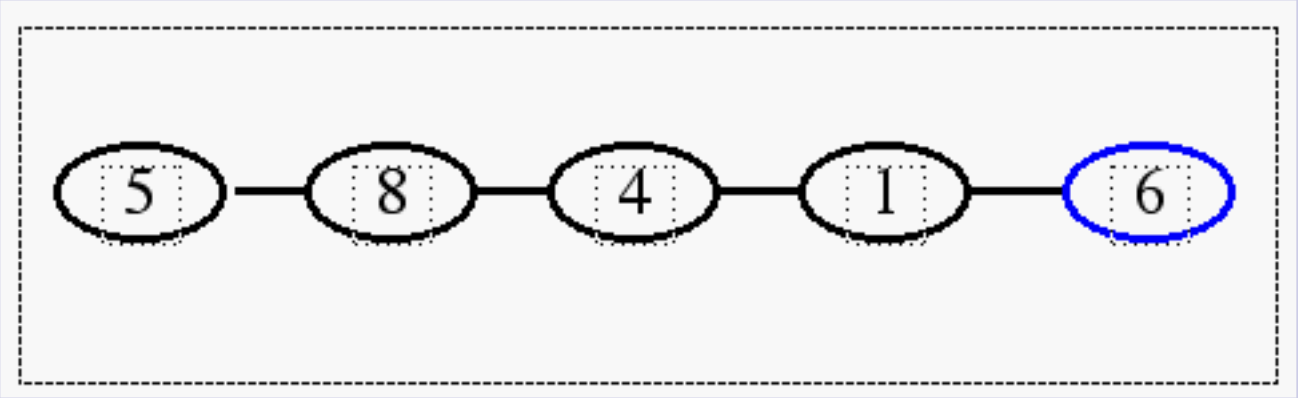
Comparators

- The most general and reusable form of a priority queue makes use of **comparator** objects.
- Comparator objects are external to the keys that are to be compared and compare two objects.
- When the priority queue needs to compare two keys, it uses the comparator it was given to do the comparison.
- Thus a priority queue can be general enough to store any object.
- The comparator ADT includes:

  isLessThan(a, b), isLessThanOrEqualTo(a,b), isEqualTo(a, b), isGreaterThan(a,b), isGreaterThanOrEqualTo(a,b), isComparable(a)

Implem. with Unsorted Sequence

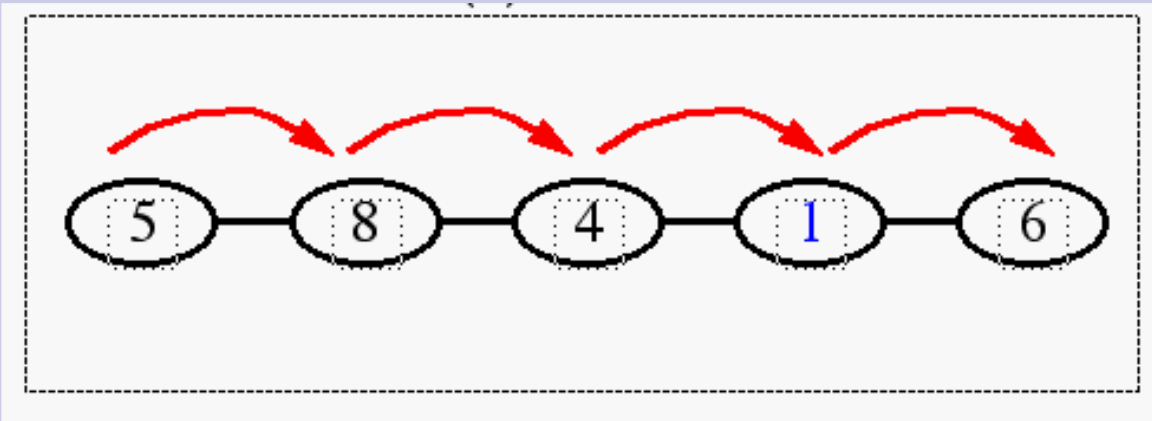
- The items are pairs (priority, element)
- We can implement insert() by using insertLast() on the sequence. This takes ***O***(1) time.



- However, because we always insert at the end, irrespective of the key value, our sequence is not ordered.

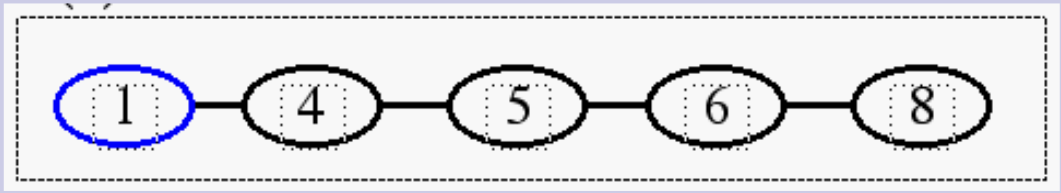
Unsorted Sequence (contd.)

- Thus, for methods such as minimum(),delete-min() we need to look at all the elements of S. The worst case time complexity for these methods is O(n).

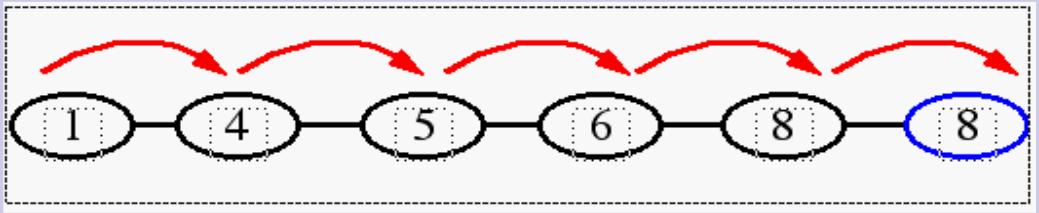


Implem. with Sorted Sequence

- Another implementation uses a sequence S, sorted by increasing priorities.
- minimum() and delete-min() take O(1) time.



- However, to implement insert(), we must now scan through the entire sequence in the worst case. Thus insert() runs in O(n) time.



Priority Queues

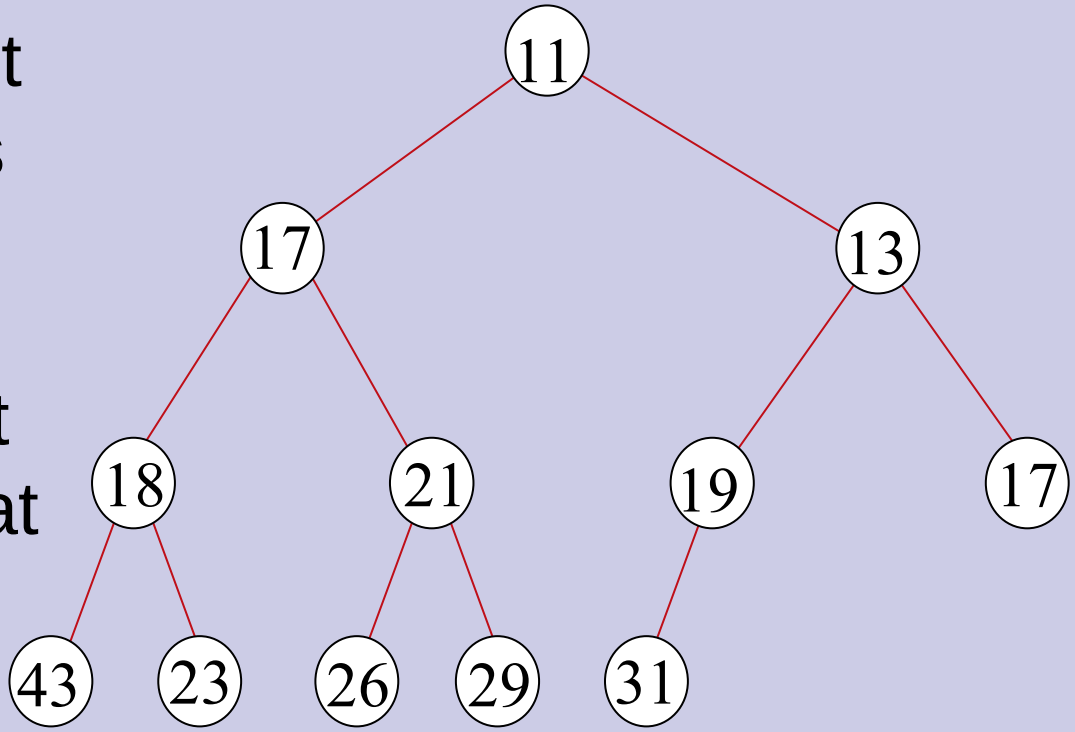
- Applications:
  - job scheduling shared computing resources (Unix)
  - Event simulation
  - As a building block for other algorithms
- A Heap *can* be used to implement a PQ

(Binary) Heaps

A binary tree that stores priorities (or priority-element) pairs at nodes

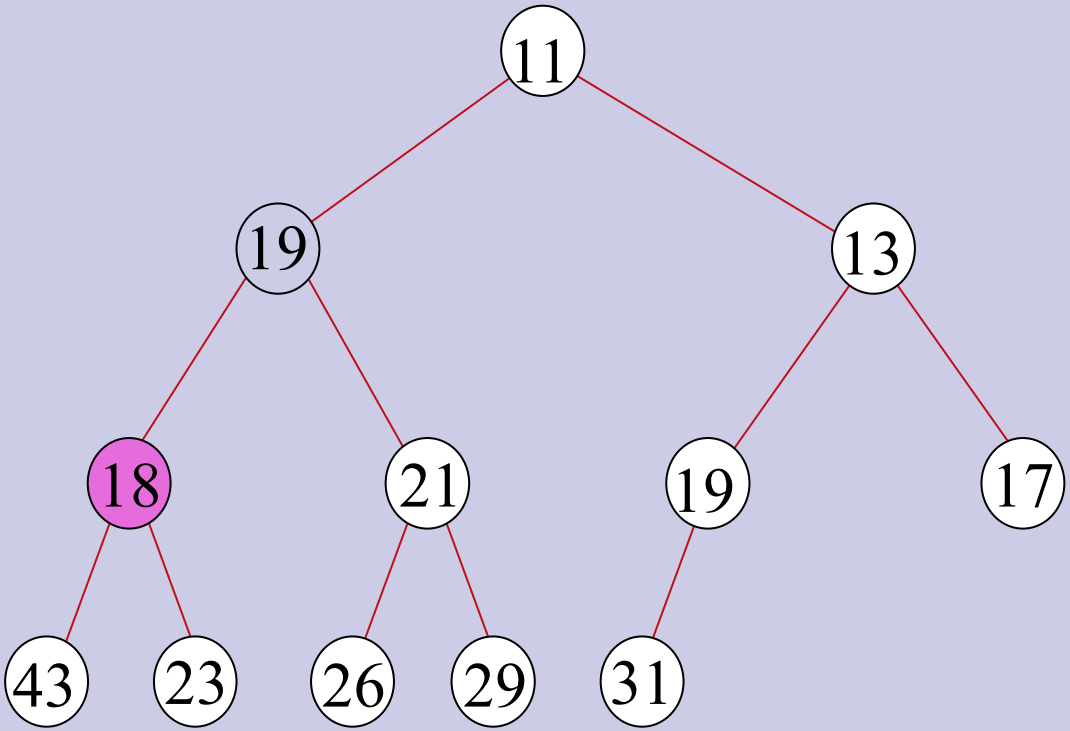
Structural property:
All levels except last are full. Last level is left-filled.

Heap property:
Priority of node is at least as large as that of its parent.



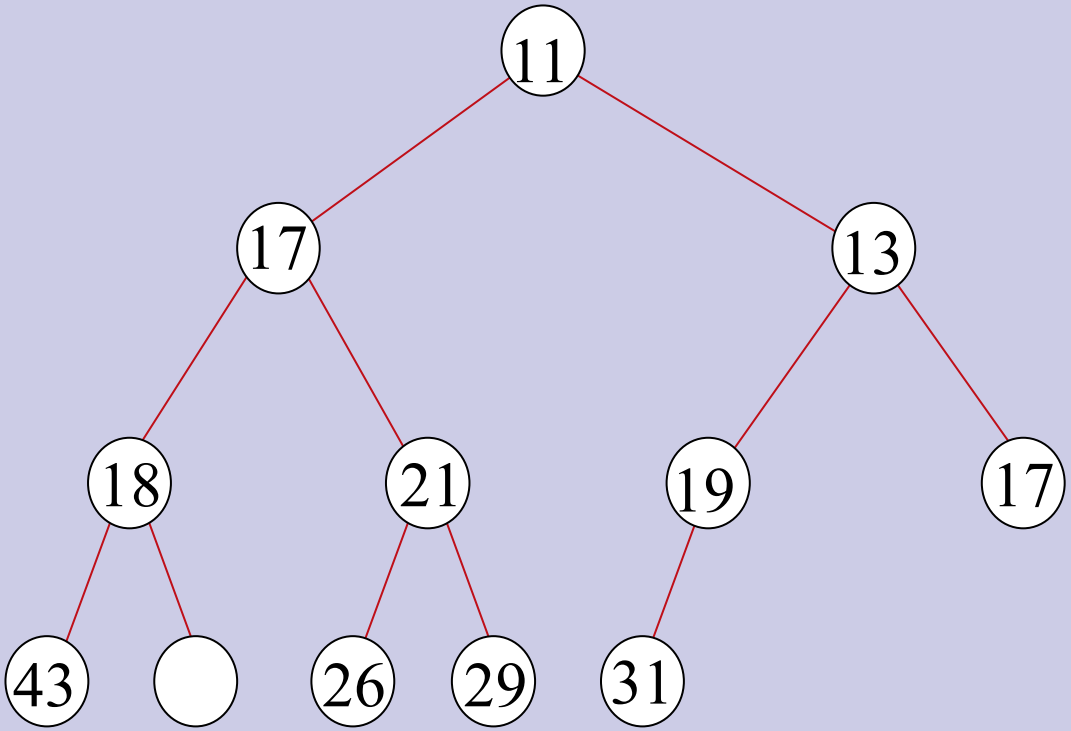
Examples of non-Heaps

- Heap property violated



Example of non-heap

- Last level not left-filled



Finding the minimum element

- The element with smallest priority always sits at the root of the heap.
- This is because if it was elsewhere, it would have a parent with larger priority and this would violate the heap property.
- Hence minimum() can be done in O(1) time.

Height of a heap

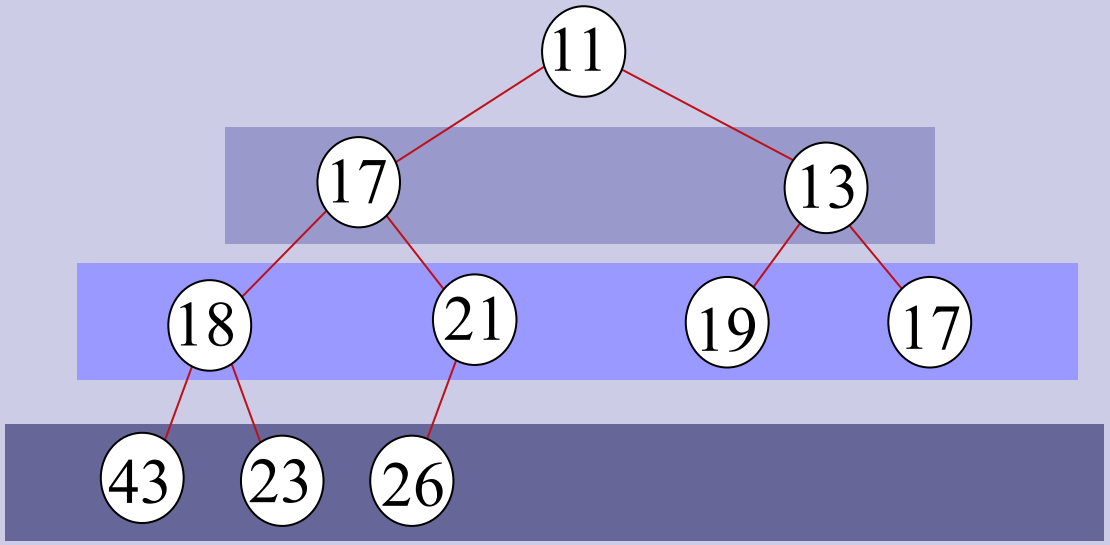
- Suppose a heap of n nodes has height h.
- Recall: complete binary tree of height h has $2^{h+1}-1$ nodes.
- Hence $2^h-1 < n <= 2^{h+1}-1$.
- $n = \lfloor \log_2 h \rfloor$

Implementing Heaps

Parent (i)
    return $\lfloor i/2 \rfloor$

Left (i)
    return 2i

Right (i)
    return 2i+1



	1	2	3	4	5	6	7	8	9	10
A	11	17	13	18	21	19	17	43	23	26

Level: 0 1 2 3

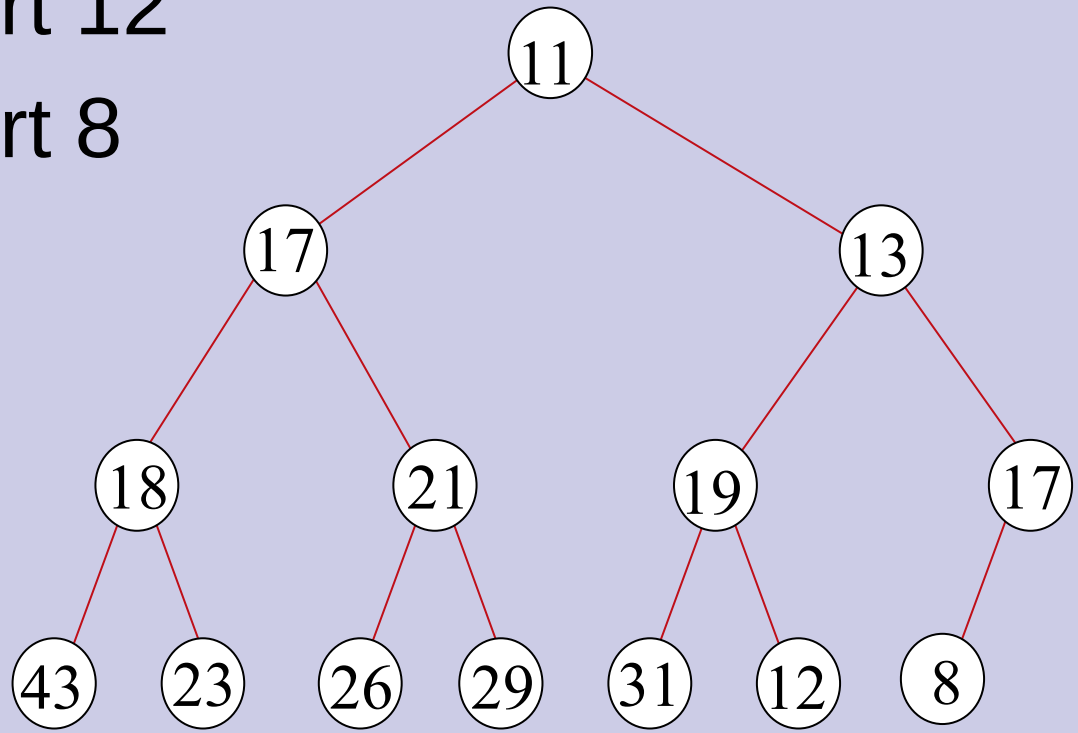
Heap property: *A*[Parent(*i*)] <= *A*[*i*]

Implementing Heaps (2)

- Notice the implicit tree links; children of node *i* are 2*i* and 2*i*+1
- Why is this useful?
  - In a binary representation, a multiplication/division by two is left/right shift
  - Adding 1 can be done by adding the lowest bit

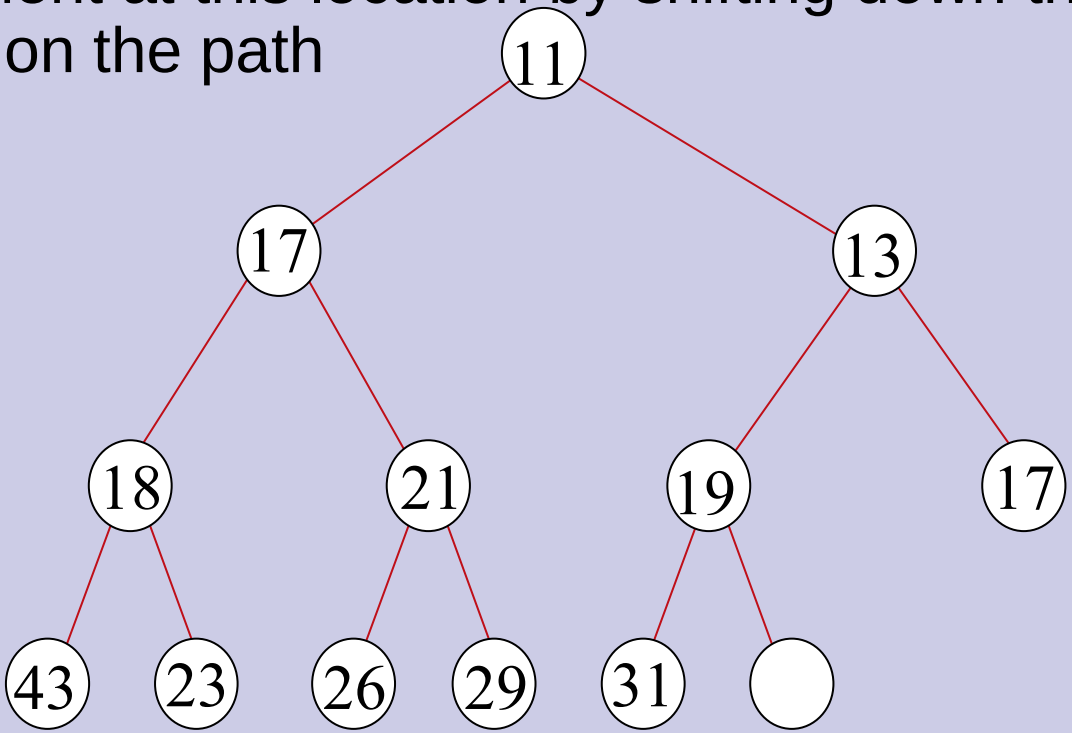
Insertion in a Heap

- Insert 12
- Insert 8



Another View of Insertion

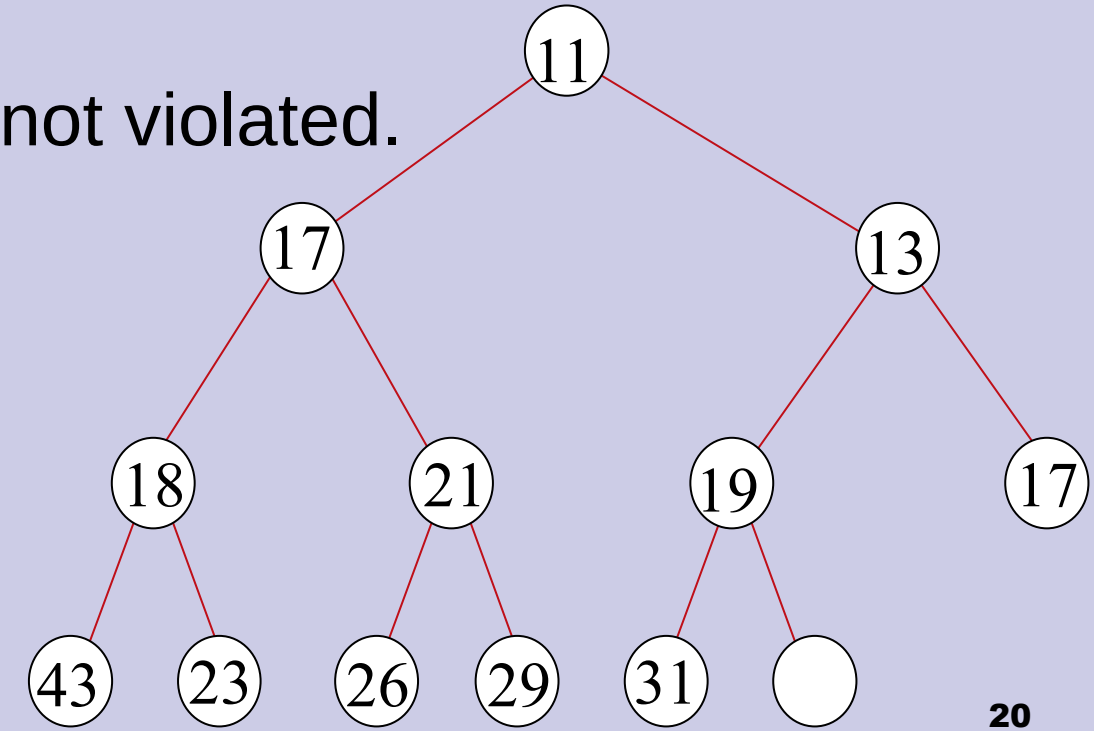
- Enlarge heap
- Consider path from root to inserted node
- Find topmost element on this path with higher priority that that of inserted element.
- Insert new element at this location by shifting down the other elements on the path



Correctness of Insertion

- The only nodes whose contents change are the ones on the path.
- Heap property may be violated only for children of these nodes.
- But new contents of these nodes only have lower priority.
- So heap property not violated.

12

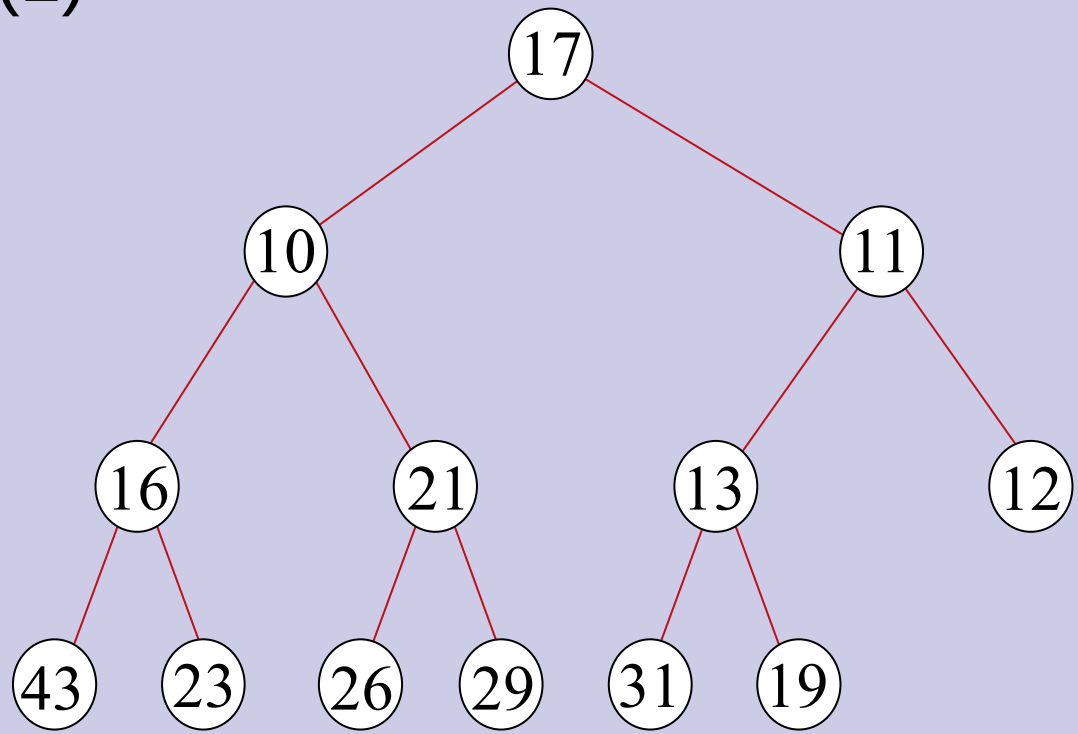


Heapify

- *i* is index into the array *A*
- Binary trees rooted at Left(*i*) and Right(*i*) are heaps
- But, *A*[*i*] might be smaller than its children, thus violating the heap property
- The method **Heapify** makes binary tree rooted at *i* a heap by moving *A*[*i*] down the heap.

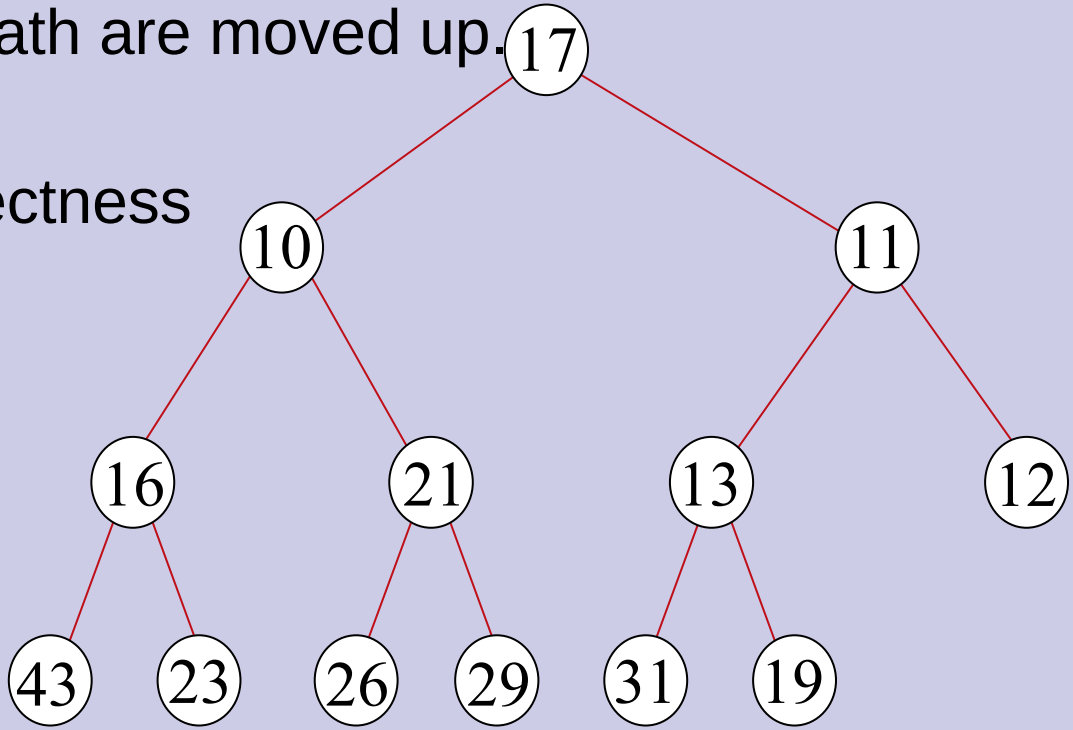
Heapify

- Heap property violated at node with index 1 but subtrees rooted at 2, 3 are heaps.
- heapify(1)



Another View of Heapify

- Heapify(i) traces a path down the tree.
- Last node on path (say j) has both A[left(j)], A[right(j)] are larger than A[i]
- All elements on path have lower priority than their siblings.
- All elements on this path are moved up.
- A[i] goes to location j.
- This establishes correctness



Running time Analysis

- A heap of n nodes has height O(log n).
- While inserting we might have to move the element all the way to the top.
- Hence at most O(log n) steps required.
- In Heapify, the element might be moved all the way to the last level.
- Hence Heapify also requires O(log n) time.