# Case Study: Searching for Patterns

Problem: find all occurrences of pattern $P$ of length $m$ inside the text $T$ of length $n$.

$\Rightarrow$ *Exact matching problem*

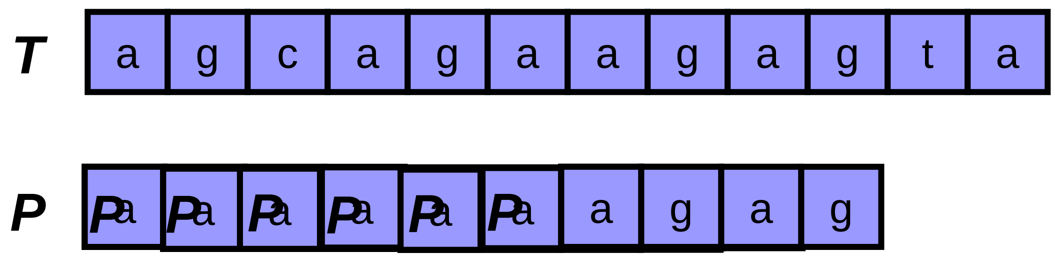
# String Matching - Applications

- Text editing
- Term rewriting
- Lexical analysis
- Information retrieval
- And, bioinformatics

# Exact matching problem

Given a string ***P*** (pattern) and a longer string ***T*** (text).
Aim is to find all occurrences of pattern ***P*** in text ***T***.

## The naive method:



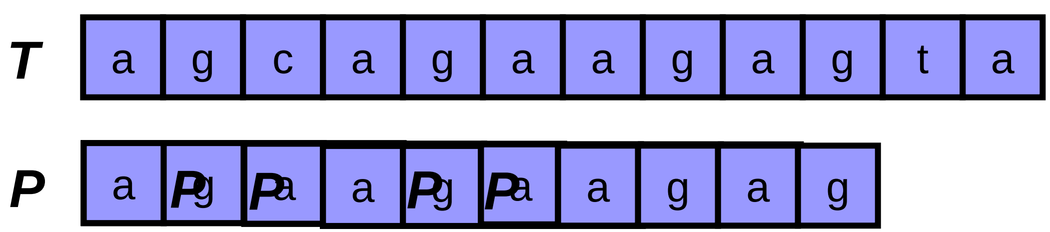

If m is the length of ***P***, and n is the length of ***T***, then

Time complexity = $O(m.n)$,

Space complexity = $O(m + n)$

# Can we be more clever ?

- When a mismatch is detected, say at position *k* in the pattern string, we have already successfully matched *k*-1 characters.
- We try to take advantage of this to decide where to restart matching



# The Knuth-Morris-Pratt Algorithm

Observation: when a mismatch occurs, we may not need to restart the comparison all way back (from the next input position).

What to do:

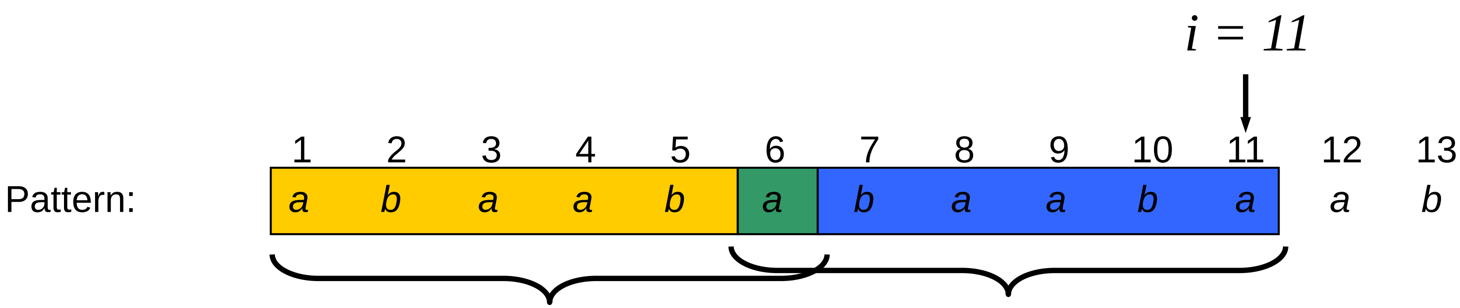
Constructing an array *h*, that determines how many characters to shift the pattern to the right in case of a mismatch during the pattern-matching process.

# KMP (2)

The **key idea** is that if we have successfully matched the prefix P[1…i-1] of the pattern with the substring T[j-i+1,…,j-1] of the input string and P(i) $\neq$ T(j), then we do not need to reprocess any of the suffix T[j-i+1,…,j-1] since we know this portion of the text string is the prefix of the pattern that we have just matched.

# The failure function h

For each position $i$ in pattern $P$, define $h_i$ to be the length of the longest proper suffix of $P[1,\ldots,i]$ that matches a prefix of $P$.




Hence, $h(11) = 6$.

If there is no proper suffix of $P[1,\ldots,i]$ with the property mentioned above, then $h(i) = 0$

# The KMP shift rule

The first mismatch in position $k=12$ of $T$ and in pos. $i+1=12$ of $P$.

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***T:*** | a | b | a | a | b | a | b | a | a | b | a | c | a | b | a | a | b | a | b | a | a | b |
| ***P:*** | a | b | a | a | b | a | b | a | a | b | a | a | b | | | | | | | | | |
| ***P:*** | | | | | | a | b | a | a | b | a | b | a | a | b | a | a | b | | | | |

$h_{11} = 6$

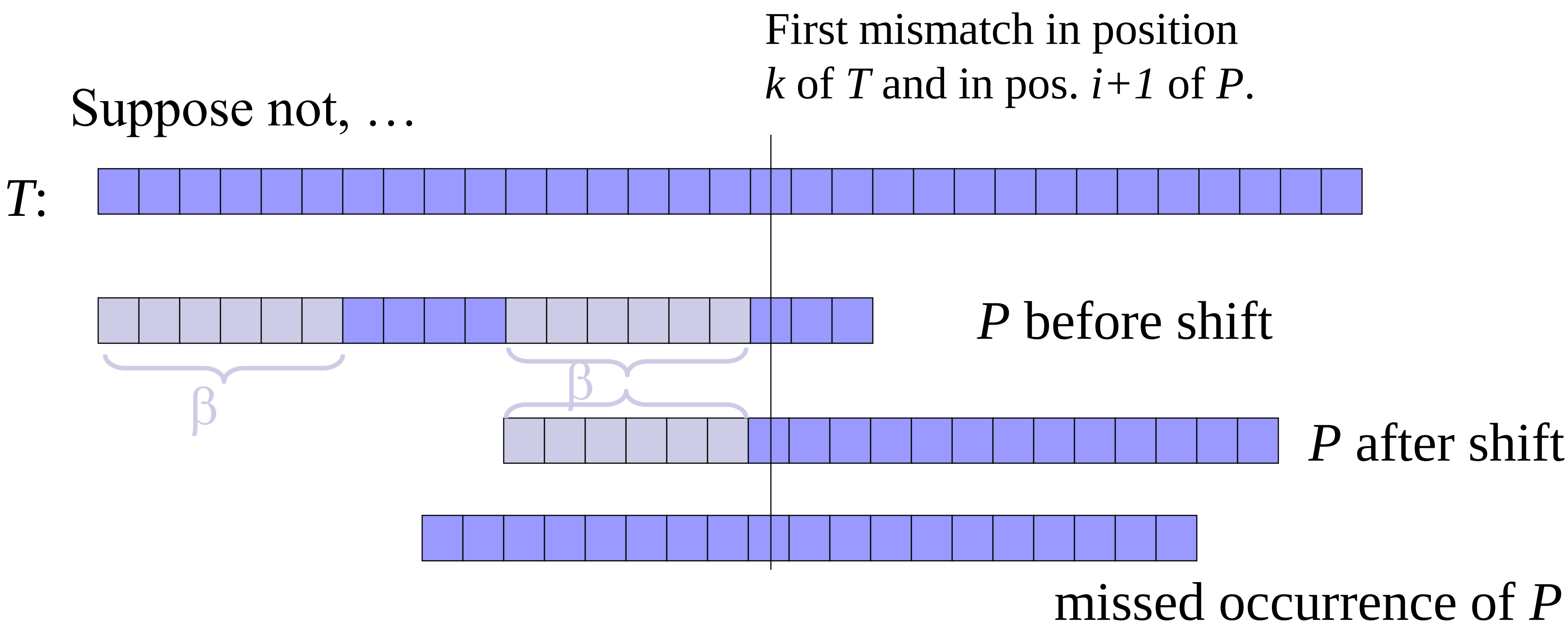
Shift P to the right so that P[1,…,h(i)] aligns with the suffix T[k- h(i),…,k-1].

They must match because of the definition of *h*.

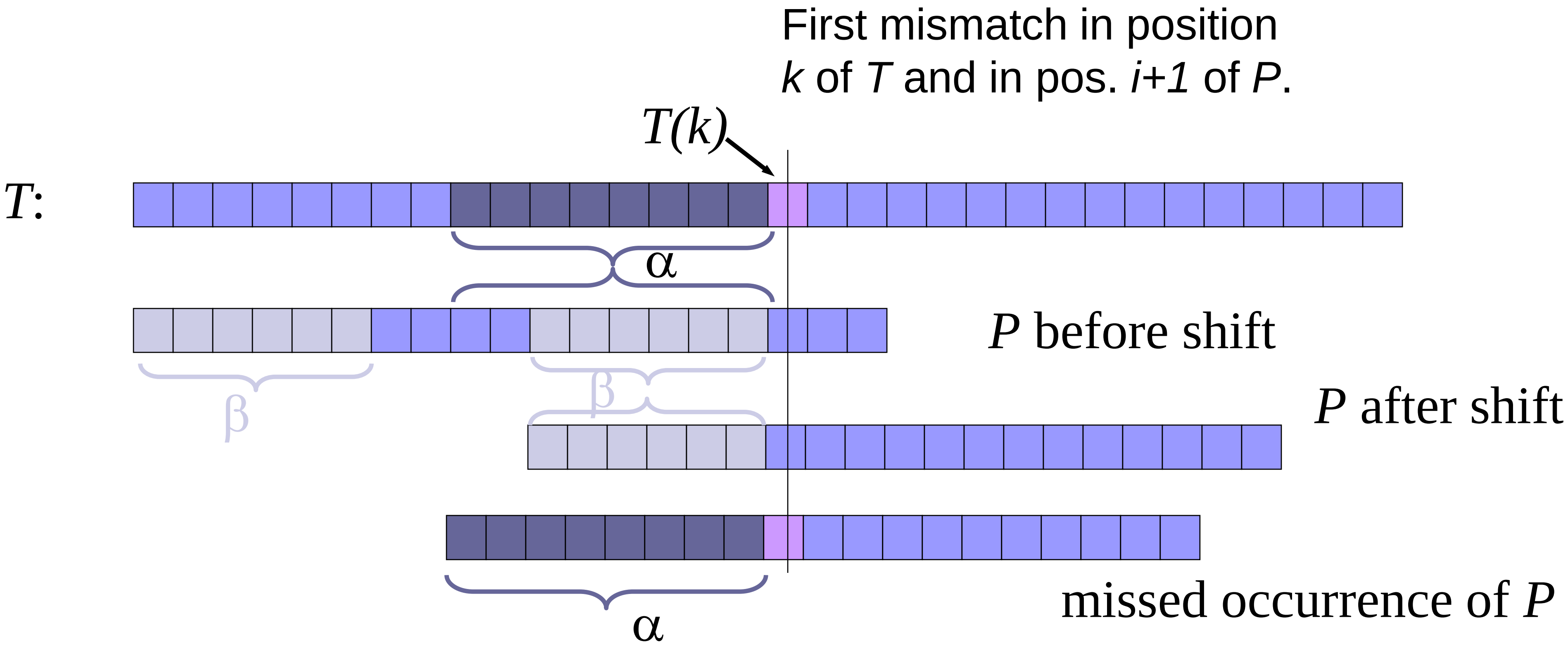
In other words, shift *P* exactly *i – h(i)* places to the right.

If there is no mismatch, then shift *P* by *m – h(m)* places to the right.

# The KMP algorithm finds all occurrences of *P* in *T*.



# Correctness of KMP.




$|\alpha| > |\beta| = h(i)$

It is a contradiction.

# An Example

**Given:**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pattern: | *a* | *b* | *a* | *a* | *b* | *a* | *b* | *a* | *a* | *b* | *a* | *a* | *b* |
| Array *h*: | 0 | 0 | 1 | 1 | 2 | 3 | 2 | 3 | 4 | 5 | 6 | 4 | 5 |

**Input string:** abaababaabacabaababaabaab.

**Scenario 1:**

$i+1 = 12$

a b a a b a b a a b a a b

a b a a b a b a a b a c a b a a b a b a a b a a b

$k = 12$

What is $h(i)= h(11) = ?$

$h(11) = 6 \Rightarrow i - h(i) = 11 - 6 = 5$

**Scenario 2:**

$i = 6 , h(6)= 3$

$i+1$

a b a a b a b a a b a a b

a b a a b a b a a b a c a b a a b a b a a b a a b

$k$

# An Example

**Scenario 3:** *i = 3, h(3) = 1*

```
                    i+1
                     ↓
              a b a a b a b a a b a a b
a b a a b a b a a b a c a b a a b a b a a b a a b
                       ↑
                       k
```

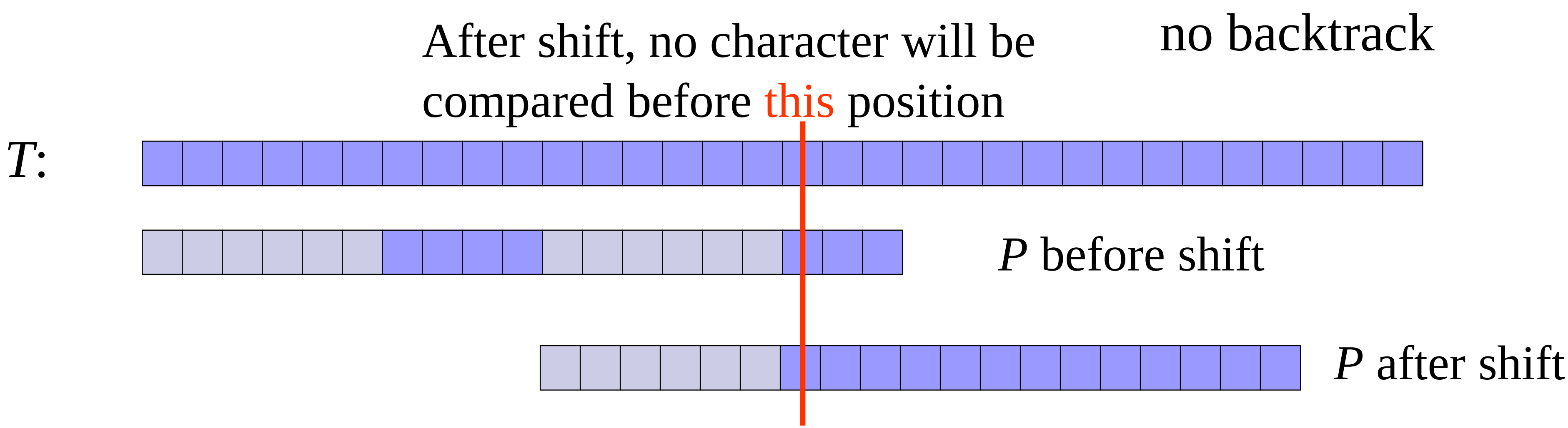
**Subsequently** *i = 2, 1, 0*

**Finally, a match is found:**

```
                        i+1
                         ↓
                        a b a a b a b a a b a a b
a b a a b a b a a b a c a b a a b a b a a b a a b
                        ↑
                        k
```

# Complexity of KMP

In the KMP algorithm, the number of character comparisons is at most *2n*.




In any shift at most one comparison involves a character of *T* that was previously compared.

Hence #comparisons $\le$ #shifts + $|T| \le 2|T| = 2n$.

# Computing the failure function

- We can compute h(i+1) if we know h(1)..h(i)
- To do this we run the KMP algorithm where the text is the pattern with the first character replaced with a $.
- Suppose we have successfully matched a prefix of the pattern with a suffix of T[1..i]; the length of this match is h(i).
- If  the next character of the pattern and text match then h(i+1)=h(i)+1.
- If not, then in the KMP algorithm we would shift the pattern; the length of the new match is h(h(i)).
- If the next character of the pattern and text match then h(i+1)=h(h(i))+1, else we continue to shift the pattern.
- Since the no. of comparisons required by KMP is length of pattern+text, time taken for computing the failure function is O(n).

# Computing h: an example

| **Given:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Failure function *h*: | 0 | 0 | 1 | 1 | 2 | 3 | 2 | 3 | 4 | 5 | 6 | 4 | 5 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Text: | *$* | *b* | *a* | *a* | *b* | *a* | *b* | *a* | *a* | *b* | *a* | *a* | *b* | |
| Pattern: | | | | | | *a* | *b* | *a* | *a* | *b* | *a* | *b* | | h(11)=6 |
| Pattern | | | | | | | | | *a* | *b* | *a* | *a* | *b* | h(6)=3 |

- h(12)= 4 = h(6)+1 = h(h(11))+1
- h(13)= 5 = h(12)+1

# KMP - Analysis

- The KMP algorithm never needs to backtrack on the text string.

***preprocessing*** ***searching***

Time complexity = $O(m + n)$

Space complexity = $O(m + n)$,

where $m = |P|$ and $n = |T|$.